Consistent Index for Timberland Capital Productivity

Discussion Paper

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Abstract

The measurement and decomposition of a timberland capital productivity as an indicator for profitability of timberland investment is considered. Due to the growing timberland industry and the accumulation of capital in international afforestation, reforestation and silviculture investments, the timberland profitability and its components perform as increasingly important benchmarks in forest policies as well as economic decision making. We show that the measurement of timberland capital productivity forms a classical index number problem. Existing timberland profitability and capital productivity indicators are reviewed and discussed. It turns out that neither index number theory nor the results established in the field especially since the 1970s have been previously accounted for when addressing the measurement of timberland profitability. In particular, we examine the Metla Index published in Finland and show that the index represents a sample of a larger set of indices which are theoretically equally valid. Since the existing index number formulas currently applied for the measurement timberland capital productivity do not comply with the widely established criteria of consistency in aggregation, the decompositions of profitability measures tend to be at least to some extent arbitrary with there being several permutations of possible decompositions. We therefore introduce a Montgomery-Vartia (MV) type of timberland capital productivity index which is consistent in aggregation. Whilst the decomposition of MV index is exact at all levels of aggregation, we present consistent sub-aggregate formulas including prices, net biological growth, harvesting and costs.

JEL Classification: C43, Q23.

Keywords: Timberland, index number theory, capital productivity, profitability.

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1. Background

Profitability of timberland investment is considered. The productivity of capital employed is becoming an increasingly important argument for forestry decision making. First, normal market-based criteria is required to guide timberland management and capital allocation in view of on-going trend of loosening-up of governmental regulation and liberalization of forest policies as well as emerging forest uses (Uotila 2005, MMM 2013; Hetemäki and Hänninen 2013). Second, capital productivity perspectives are gaining weight since afforestation and reforestation activities are playing a growing role in global timber production implying allocation of substantial capital and investment in the sector (FAO 2010); see also Indufor (2012). Finally, theoretically robust profitability measures are required for empirically sound and reliable timberland enterprise benchmarking (e.g. Viitala 2010).

Our perspective to the timberland capital productivity is based on index number theory[1]. Two main results will be introduced. First, an exact decomposition for the index formula applied by Finnish Forest Research Institute ("Metla Index") will be examined for various applied index formulas. Problems related to the consistency of Metla Index will be elaborated and discussed. Second, we establish an index for timberland capital productivity which is consistent in aggregation. The paper is largely based on the M.Sc. thesis by Äärilä (2013) but also some new results, observations and remarks will be presented.

Because indices compress data, certain problems appear related to the consistency and resulting from the calculation methods applied for the numbers. It is seldom possible to express something intricate with simple index number without losing some aspects of the reality in the process. Depending on the methodology, shortcomings vary but are unavoidable. Index construction as it is, is a field of trade-offs where no comprehensive or exactly right methods exist. The question is which of these shortcomings can be accepted.

Recent fundamental results of index number theory especially from the 1970s onwards have been only very scarcely accounted for when considering measures for timberland capital productivity. Afriat (1972) and Samuelson and Swamy (1974) argued against the typical homothecity assumption

[1] "Profitability" and "capital productivity" are used here as synonyms; we prefer the term "capital productivity" as it more precisely describes the essence of the measurement mission from the viewpoint of economics and index number theory.

of the economic approach to index numbers[2]. They concluded similarly that one should aim for more general and thus less restricted index forms that can approximate wider variety of aggregator functions. Nevertheless, Samuelson and Swamy (1974) mention that homothecity is not so unrealistic of an assumption in production theory where some rather well behaving returns to scale often exist. The research at the time showed that nonparametric methods impose an implicit structure on the technology. It was shown that Laspeyres or Paasche indices imply either Leontief preferences (technology) for which there is no substitution, or linear preferences (technology) in which all inputs are perfect substitutes (Diewert 1976).

Sato (1976) and Vartia (1976) both came up with the log-change Sato-Vartia (Vartia II) index. Sato (1976) derived the index as being exact with the CES-production function. At the same time and independently, Vartia (1976) formulated the same index as a descriptive formula satisfying certain desired mathematical properties[3]. During the same period Vartia (1976) advocated the notion of *consistency in aggregation*, a property the Sato-Vartia index does not have.

Definition 1. Consistency in Aggregation ***(Vartia 1976; see also Diewert 1978):***
"This requirement states that if one calculates the index for the larger aggregate in two steps, calculating first the indices for the subaggregates and then feeding these along with the value data of the subaggregates into the same formula, one must necessary get the same result as if one had calculated the index in one step" (Pursiainen 2005).

The Montgomery-Vartia (Vartia I) index, another descriptive index proposed by Vartia (1976), satisfies the condition for consistency in aggregation; it is also exact with the C-D- production function. Diewert (1976) established the concept of *superlative indices* by calling such index formulas superlative that are exact with flexible functional forms approximating arbitrary twice differentiable linearly homogenous functions to the second order. Superlativity is regarded often as a desirable property by the economic approach because only an approximation for the linearly homogenous aggregator is required and homogenous aggregators are many in economics. While the originally descriptive Fisher's Ideal and Walsh index turned out to be superlative in respect to quadratic function forms, Diewert (1976) proved that the Törnqvist index is exact with the translog function making it very

[2] Economic approach, also known as the functional approach, assumes that there exists a relation between price and quantity and it pursues index formulas that are in line with the indifference curves defining this assumed relation.
[3] Descriptive approach to index numbers treats quantity and price as independent variables and hence it relies on index tests and indices are assessed purely based on their ability to satisfy these tests (see e.g. Fisher 1922). The approach is also known as axiomatic, statistical, test and atomistic approach.

useful in economic applications. In practice superlativity means that the indices treat prices and quantities between the different periods equally and can account for substitution in the framework of homothetic preferences. Indices that are exact with more restricted functional forms and therefore approximate the superlative indices Diewert (1978) named pseudo-superlative. In particular, Diewert (1978) showed that the Montgomery- Vartia price index approximates any superlative price index to the second order around an equal price and quantity point.

Replying to Vartia's (1976) definition of consistency in aggregation, Diewert (1978) acknowledged that the superlative indices are only approximately consistent in aggregation when the pseudo-superlative Montgomery-Vartia is exactly consistent. Vartia (1983) proved that when formulating index formulas in algorithmic form arbitrary general demand functions start to converge into those algorithms. Therefore no assumptions regarding the underlying indifference curves need to be made because they all converge into algorithmic forms which can be seen to weaken the argument for specific exact functional forms. Balk (2003) extended the traditional two factor index number problem to cases where more than two factors exist and concluded that Sato-Vartia and Montgomery-Vartia indices seem to scope the best in that context. As economic indices are often seen as hierarchic aggregations, Pursiainen (2005, 2008a, b) argue with his quasilinear presentation that indices that are consistent in aggregation should be preferred. Consistent aggregation is in line with a hierarchic structure as it is able to provide a simple and easily interpretable method even for complex systems.

The consistency in aggregation is a very desirable property in timberland capital productivity measurement where not only the value aggregates but also their decompositions[4] are often utilized and highly valued in practice. On the other hand, the lack of exact consistency in aggregation makes the use of many widely applied superlative indices problematic here. The index numbers by a consistent index formula are easy to comprehend and the users of the index do not have to understand index number theory in order to fully utilize the indices. With some other index formulas major problems can emerge if the users do not understand the implications the applied formulas have to the utilization of sub-indices and what is their relationship to the aggregate.

Consistent aggregation would enable the exact mathematical decomposition of the timberland capital productivity into its decomponents while still using the same formula in all the stages of the calculation. The possibility to track different decomponents separately helps investors and forest owners to

[4] Contribution of timber prices, growth and harvest volumes and costs.

study and observe the investment prospects of timberlands in detail and to act accordingly. For policy makers the sub-indices can operate as guiding indicators and a feedback mechanism for more precise legislation and that way steer forest policies towards more sustainable and efficient direction.

To conclude, we shall present our desiderata for a timberland capital productivity measure – it is rather simple. The index model has to be **consistent** and **exact**. By consistent we refer to consistency in aggregation and non-arbitrariness of the index formulas. By exact we mean that the decomposition from aggregate can be done analytically in proportion and no statistical error terms are present[5][6].

2. Review of applied measures for timberland profitability

The NCREIF Timberland Index (TI) is published by National Council of Real Estate Investment Fiduciaries. The index can be seen as the leading timberland return index in the US and it is widely utilized in literature. The Index aims to track quarterly and annual returns of large pool of US-based investment-grade timberlands owned by tax-exempt institutions. The fundamental idea behind the composition of the NCREIF Timberland Index is that timberland investment managers contribute property return data to NCREIF, which then calculates the aggregate index (NCREIF 2013). The index formulas are as follows.

$$\text{Income return} = \frac{\mathrm{EBITDDA_t}}{\mathrm{MV_{t-1}} + 0.5(\mathrm{CI_t} - \mathrm{PS_t} + \mathrm{PP_T} - \mathrm{EBITDDA_t})} \tag{1}$$

$$\text{Capital return} = \frac{\mathrm{MV_t} - \mathrm{MV_{t-1}} - \mathrm{CI_t} + \mathrm{PS_t} - \mathrm{PP_t}}{\mathrm{MV_{t-1}} + 0.5(\mathrm{CI_t} - \mathrm{PS_t} + \mathrm{PP_T} - \mathrm{EBITDDA_t})} \tag{2}$$

For the most part $\mathrm{EBITDDA_t}$ includes the operating revue resulting from timber sales, but it can also include some smaller revenue streams such as sales of hunting leases. EBITDDA is reported as gross of management and fund-level fees. $\mathrm{MV_t}$ is the market value of the property at the end of quarter t. $\mathrm{CI_t}$ is the capital improvements during the period. $\mathrm{PS_t}$ represents proceeds from partial sales of land and $\mathrm{PP_t}$ the costs from partial purchases. The return of a single property is weighted by its market

[5] Mathematically we require indices to satisfy the following condition $\mathrm{P_0^1 Q_0^1} = \frac{\mathrm{V_1}}{\mathrm{V_0}}$

[6]Term "exact" should not be confused here with Diewert's (1976, 1978) concept of superlative indices as they sometimes are referred in literature as exact indices because they are exact with function forms approximating some arbitrary aggregator function to the second order.

value. The coefficient of 0.5 illustrates the Dietz (1966) method's assumption that capital improvements, sales, purchases and proceeds occur in the middle of the period.

Due to NCREIF TI's central role in timberland returns measurement its calculation method has been examined and various researches have pointed out some weaknesses in the index. Binkley et al. (1996) made a remark about the possible appraisal smoothing bias in the index which is expected to lead to less volatile and lagged price movements and smoother return series. Later, Scholtens and Spierdijk (2010) considered the appraisal smoothing bias so severe that they tried to correct it by artificially increasing the volatility of the index and received less flattering results regarding the diversification benefits of timberland investments. More troubles are caused by the density of appraisals. Most of the properties are appraised at the end of the fourth quarter, decreasing the usefulness of the quarterly return figures and making the annual returns the most valid (Hancock Timber Resource Group 2003a). Aronow et al. (2004) mention the problem originating from the change of properties within the pool. It is hard to know whether a change in income or capital return or in property value is a result caused by a change in fundaments or a change in the distribution of the timberland holdings. Lutz (2008) reported that the contribution of the fourth quarter to total appreciation returns have steadily increased and he concludes that the annual appraisals create factitious fluctuations into the quarterly appreciation series. To better capture the returns an institutional investor could receive the NCREIF introduced the **NCREIF Timber Fund and Separate Account Index (TFSAI)** in 2012. The index is simply calculated by merging the time-weighted returns of timber funds and separate accounts reported by managers. Unlike TI, the index acknowledges leverage, fees and cash.

The John Hancock Timber Index (JHTI) is a synthetic index constructed by John Hancock Timber Resource Group to express the returns from timberlands before the year 1987, from which on the NCREIF Timberland Index starts to report returns (Binkley et al 2006).

$$\text{Rate of return}_{\mathrm{t}} = \frac{\text{Net income}_{\mathrm{t}} + \text{Capital value}_{\mathrm{t}}}{\text{Capital value}_{\mathrm{t-1}}} - 1 \qquad (3)$$

According to Hancock Timber Resource Group (2003b) the equation represents the returns from a fully-regulated forest. The formula assumes that the costs are a relative proportion of the forest value and can be incorporated into the income rate coefficient. JHTI defines the net income as a function of income rate and price.

$$\text{Net income}_t = \text{Income rate} \times P_t \tag{4}$$

Income rate is an estimate of the average cash flow from forest in relation to the asset value which is represented by the price P_t. The Income rate is estimated using historical data of the relationship and is essential for the result of the entire index equation as Hancock Timber Resource Group (2003b) puts it: *"The choice of income rate has a direct influence on the level of historical timberland return estimates produced by the John Hancock Timber Index. Its specification is a matter of informed judgment."*

The capital value in equation 5 is weighted average price of the past eight periods, while the weights decrease when moving further back in time.

$$\text{Capital value}_t = \frac{8P_t + 7P_{t-1} + 6P_{t-2} + 5P_{t-3} + 4P_{t-4} + 3P_{t-5} + 2P_{t-6} + P_{t-7}}{36} \tag{5}$$

Price variable P_t, which is proxy for timberland value, is a weighted average price of the different timber assortments and species. By using this formula the JHTI assumes that the value of timberland and growing stock follow the witnessed timber prices, which means that future price expectations are assumed to be based on current and past price levels.

Timberland Performance Index (TPI) was an index that tracked timberland fund performance. It did not track returns of timberlands investments directly instead it tracked the returns of US-based funds that owned timberland (Caulfield 1994). The index was based on public return figures reported by the funds to *Real Estate Profiles*, a publication published by Evaluation Associates. Due to the new arrivals into timberland investing that did not report their fund returns to *Real Estate Profiles*, in 1992 the index's coverage of US timberland fund assets dropped from 100% to 25% (Caulfield 1994). The aggregate index returns are calculated as weighted averages of single fund returns which were reported quarterly. The return figure is a result of the changes in fund values and hence the index formula is not able to make distinction between income and appreciation component because they are both wrapped into a single figure. The index value is calculated in period i as follows (Caulfield 1998).

$$\mathrm{TPI}_{\mathrm{i}} = \mathrm{TPI}_{\mathrm{i-1}}\left(\frac{1}{\mathrm{T}_{\mathrm{i-t}}}\sum_{\mathrm{j=1}}^{\mathrm{m}} \mathrm{R}_{\mathrm{ji}}\mathrm{W}_{\mathrm{ji-1}}\right) \tag{6}$$

$\mathrm{TPI}_{\mathrm{i}}$ is the value of the fund in period i, R_{ji} the change in value of timberland fund *j* from period *i*-1 to i. W_{ji-1} is the dollar value of the fund in i-1 and $\mathrm{T}_{\mathrm{i-1}}$ is the dollar value of all the funds in *i*-1. Caulfield (1994) mentions that the biggest downside of the index is that it cannot be broken into sub-regional indices, since funds report total returns of their diversified portfolios and hence only the aggregate returns are available. Like NCREIF TFSAI, the return figures produced by the TPI formula will contain cash, non-timberland assets and possible gearing applied by the funds.

In Finland the Finnish Forest Research Institute (Metla) publishes timberland return index that also produces separate decomposition indices for the different timberland return components. This Metla index will be discussed and elaborated in sections 4 and 5.

3. Timberland capital productivity and index number theory

Define the value of timberland (V), the timberland capital, as the product of price (P) and standing timber stock quantity (Q) over different timber assortments (i) as follows[7]:

$$\mathrm{V}_{t} = \mathrm{V}(\mathrm{P}_{\mathrm{t}}, \mathrm{Q}_{\mathrm{t}}) = \sum_{\mathrm{i=1}}^{\mathrm{n}} (\mathrm{p}_{\mathrm{t,i}}\mathrm{q}_{\mathrm{t,i}}) \tag{7}$$

where p and q denote for price and quantity of timber assortments 1,…n, respectively, and t denotes for time. In our notation capital letters stand for aggregates over timber assortments.

In the following, we consider two sequential periods, period 0 and period 1, also called as the base period and observation period. Assume a forest owner with timberland capital $\mathrm{V}(\mathrm{P}_0, \mathrm{Q}_0)$ at the end of period 0. The growth of the forest follows a general biological growth process in which the forest stock in the latter period is a function of forest stock at the end of the previous period $\mathrm{q}^{*}_{1,\mathrm{i}} = \mathrm{G}_{\mathrm{i}}(\mathrm{q}_{0,\mathrm{i}})$[8]. Hence, the timberland capital at the end of base period $\mathrm{V}_0 = \mathrm{V}(\mathrm{P}_0, \mathrm{Q}_0)$ is considered as a capital input

[7] In this paper timberland is valued to its liquidation value.
[8] G_{i} is normally assumed to have logistic form $\mathrm{G}'_{\mathrm{i}} < 0$ (see e.g. Amacher et al. 2009). However, note that our approach is purely descriptive and that we only assume that $\mathrm{G}_{\mathrm{i}}$is a function with known functional form.

for the total timberland capital to be gained at the end of observation period $V_1^* = V(P_1, Q_1^*) = V\big(P_1, G(Q_0)\big)$, where $\sum_{i=1}^{n} q_{1,i}^* = \sum_{i=1}^{n} G_i(q_{0,i}) = G(Q_0) = Q_1^*$.

Assuming harvesting volume $h_{1,i}$ during period 1, we get the standing stock at the end of period 1 $q_{1,i} = q_{1,i}^* - h_{1,i}$. Note that $\sum_{i=1}^{n} q_{1,i} = Q_1$ and $\sum_{i=1}^{n} h_{1,i} = H_1$. Denote the value of harvesting during period 1 for $P_1 H_1$[9].

Finally, denote lump sum costs for C_1[10] during period 1 (the observation period).

Hence, the net timberland capital at the end of period 1 resulting from the initial investment of V_0 into the venture by the forest owner can be established as $V_1^* - C_1 = V_1 + P_1 H_1 - C_1 = V_0 + (V_1 - V_0) + P_1 H_1 - C_1$

Figure 1 illustrates this relation between values in different periods.

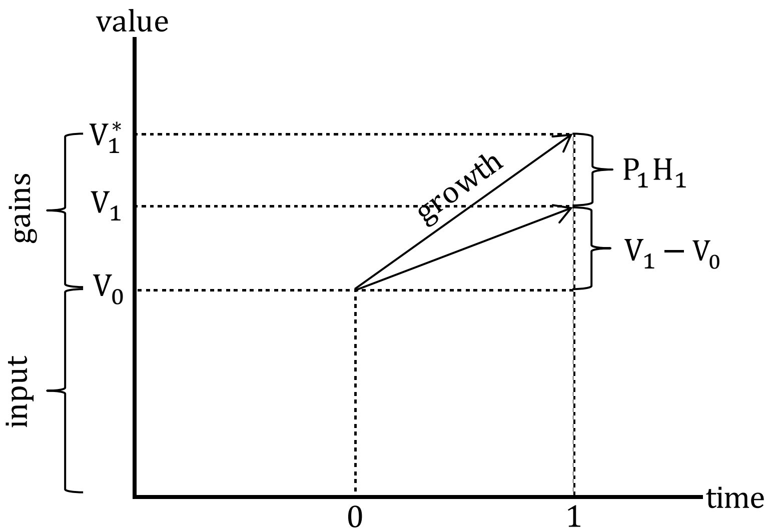

Figure 1. Development of Timberland Capital.

[9] Economic theory normally assumes that harvesting decisions are based, for example, on the forest owner maximizing the net present value of stumpage revenue (e.g. Amacher et al. 2009). In the following, we merely observe the realized harvesting at given prices. Also, V_0 is an exogenous variable.

[10] Costs are assumed to be a lump sum which is a simplification but often a necessary practice in real life statistical measurement. The sum consists of items such as silvicultural improvement work and administrational costs.

Next, define Timberland Capital Productivity as follows.

<u>Definition 2.</u> Timberland Capital Productivity (TCP)

$$TCP = \frac{V_1^* - C_1}{V_0} = \frac{\sum_{i=1}^{n}\left(p_{1,i}q_{1,i} + p_{1,i}h_{1,i}\right) - C_1}{\sum_{i=1}^{n}\left(p_{0,i}q_{0,i}\right)}$$

$$= \frac{\sum_{i=1}^{n}(p_{1,i}q_{1,i}^*)}{\sum_{i=1}^{n}(p_{0,i}q_{0,i})} - \frac{C_1}{\sum_{i=1}^{n}(p_{0,i}q_{0,i})} \qquad (8)$$

From the viewpoint of index number theory, there are two immediate observations to be noted regarding the established TCP measure. First, the difference $V_1 - V_0$ forming an implicit component in the TCP nominator captures both the effects of net (of harvesting) biological growth (volume change) denoted for $\gamma_{1,i} = q_{1,i} - q_{0,i}$, as well as the change in prices between the sequential periods. We are not only interested in TCP in absolute terms but it is also essential to be able to decompose the productivity into relative quantity[11] and price contributions. To provide a simple illustration for the index number problem present, let us write the difference $V_1 - V_0$ in combinations of Laspeyres and Paasche forms:

$$\mathrm{V}_1 - \mathrm{V}_0 = \mathrm{P}_1\mathrm{Q}_1 - \mathrm{P}_0\mathrm{Q}_0$$
$$= \mathrm{Q}_1(\mathrm{P}_1 - \mathrm{P}_0) + \mathrm{P}_0\left(\sum_{i=1}^{n}\gamma_{1,i}\right) \qquad (9)$$
$$= \mathrm{Q}_0(\mathrm{P}_1 - \mathrm{P}_0) + \mathrm{P}_1\left(\sum_{i=1}^{n}\gamma_{1,i}\right)$$

The problem with index numbers is that often one is not comparing only one variable but many at the same time. The traditional index number problem boils down here to the fundamental key question of how a shift from one observation point to another should be measured, so that the decomposition indices would provide the best possible illustration of the change, when there are more than one variable contributing to the observation. Following Diewert (2005) and Montgomery (1929), Figure 2 illustrates the index number problem graphically for a single timber assortment i.

[11] "Quantity" and "volume" are considered as synonyms in the text since volume is the quantity of enclosed three-dimensional space and timber is usually quantified in volume units.

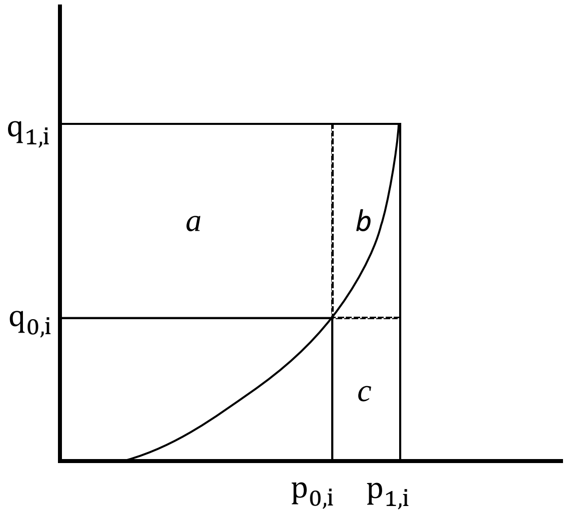

Figure 2. Montgomery´s illustration of the index number problem

Figure 2 can illustrate two different timber supply and corresponding price levels in the markets. The essence of the question is how the area b should be divided between separate price and quantity components when the capital of timber assortment i standing in the forest at the end of period 0 equaling to $p_{0,i}q_{0,i}$ changes to $p_{1,i}q_{1,i}$ at the end of period 1. Note that the total value change equals a+b+c. Following a classic illustration by Montgomery (1929) one can imagine some supply function, defining q_i as a function of p_i, drawn into the figure[12]. The theoretical definition for price index in absolute terms is that it is the integral of the function from $p_{0,i}$ to $p_{1,i}$, whereas the index number for quantity would be the integral of the inverse[13] supply curve from $q_{0,i}$ to $q_{1,i}$. Naturally, the shape of the curve is of great interest since it would establish through the integrals the decomposition of the area b into quantity and price components. Thus, the equation 9 can be interpreted representing two extreme positions. For example, the Paasche formulation for quantity in equation 9 would allocate the whole area b to quantity change whereas the Laspeyres formulation would allocate the area b into price change.

Second, it should be noted that the harvests do not have a counterpart in the base period 0. The situation can be seen as analogic to the so called *the new good problem* which is well established in index theory literature. The issue is faced by any timberland index publisher wishing to account for harvests in a separate index. As can be seen in Figure 1 above, H_1 emerges during period 1 as “a new good” which does not have counterpart embodied in V_0. Hicks (1940) addressed the new goods problem in the context of inflation calculation concluding a theoretical solution for a case where the price of the

[12]Virtually no *a priori* assumptions are here made about the elasticity of supply function. The example is generic.

[13] p_i is a function of q_i

new good is unknown in period 0. Here we would know the base period prices but the quantity is simply absent. Later, this issue will be illustrated in practice and ways to overcome it will be presented.

To summarize the discussion this far and to demonstrate its implications to timberland capital productivity measurement we conclude that a desirable index should have the following consistent and exact structure[14].

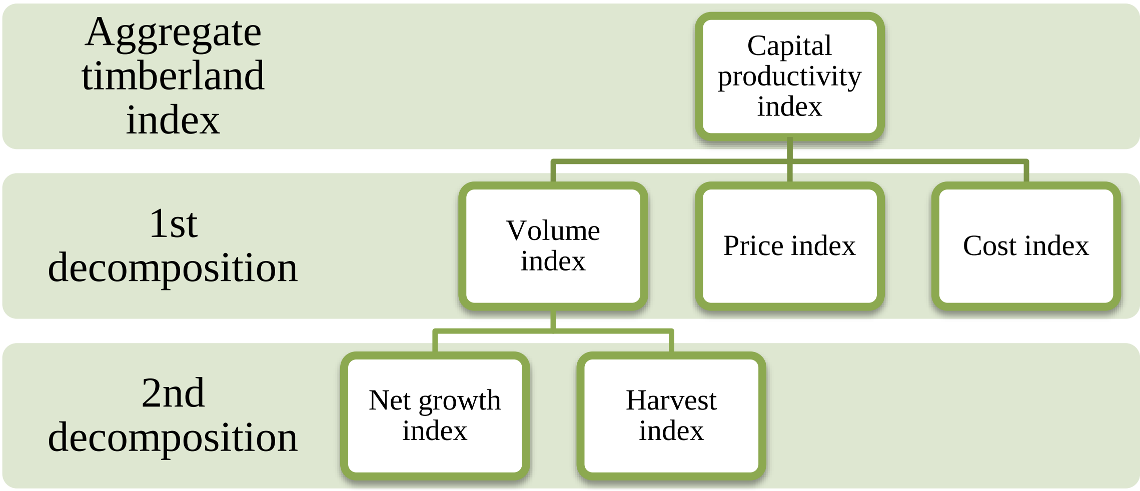

Figure 3. A desirable structure for timberland capital productivity index.

4. The Metla index

In view of the discussion in previous sections, we will in the following discuss in a greater detail a timberland capital productivity measure which is applied in Finland for regular publishing of timberland profitability[15]. This particular log-change[16] formula ("The Metla Index") is utilized by the Finnish Forest Research Institute (FFRI) in their official reporting and its development is discussed in Lausti and Penttinen (1998) and Penttinen and Lausti (2004). In addition to statistical bulletins and publications by FFRI, the index has been utilized in scientific research (e.g. Lausti 2004, Penttinen and Lausti 2009, Viitala 2010). The Metla Index is defined as follows.

[14] The structure of the forest can affect the index structure, e.g. net growth index is redundant in fully-regulated forest.
[15] The index is published annually in the *Finnish Statistical Yearbook of Forestry* which is a part of *Official Statistics of Finland*. The index series is also available online at MetINFO (Finnish Forest Research Institute 2013).
[16] For the properties of logarithmic change one should see Törnqvist et al. 1985. Here we mention symmetricity and additivity of the change.

$$\mathrm{TCP_{Metla}} = \ln\left(\frac{\sum_{i=1}^{n} p_{1,i}(q_{0,i} + \gamma_{1,i}) + \sum_{i=1}^{n} p_{1,i}h_{1,i} - C_1}{\sum_{i=1}^{n} p_{0,i}q_{0,i}}\right) \tag{10}$$

If we substitute $(q_{0,i} + \gamma_{1,i} + h_{1,i})$ with our gross timber stock variable $q_{1,i}^*$ and separate the cost term, we can write the formula as follows.

$$\ln\left(\frac{\sum_{i=1}^{n} p_{1,i}q_{1,i}^* - C_1}{\sum_{i=1}^{n} p_{0,i}q_{0,i}}\right) = \ln\left(\frac{\sum_{i=1}^{n} p_{1,i}q_{1,i}^*}{\sum_{i=1}^{n} p_{0,i}q_{0,i}} - \frac{C_1}{\sum_{i=1}^{n} p_{0,i}q_{0,i}}\right)$$

$$= \ln\left[\frac{\sum_{i=1}^{n} p_{1,i}q_{1,i}^*}{\sum_{i=1}^{n} p_{0,i}q_{0,i}} \times \left(1 - \frac{C_1}{\sum_{i=1}^{n} p_{1,i}q_{1,i}^*}\right)\right] \tag{11}$$

$$= \ln\left(\frac{\sum_{i=1}^{n} p_{1,i}q_{1,i}^*}{\sum_{i=1}^{n} p_{0,i}q_{0,i}}\right) + \ln\left(1 - \frac{C_1}{\sum_{i=1}^{n} p_{1,i}q_{1,i}^*}\right) = \ln\left(\frac{V_1^*}{V_0}\right) + \ln\left(1 - \frac{C_1}{V_1^*}\right)$$

Thus, the formula reduces now down into already familiar aggregate value ratio and to a separate cost index.

Next we ignore the already separated lump sum cost index that will always remain fixed and concentrate on the value ratio. It can be shown that one way to decompose the value index exactly is to first split it arbitrarily either into logarithmic Paasche's price and logarithmic Laspeyres' volume indices or alternatively into logarithmic Laspeyres' price and logarithmic Paasche's volume indices. This corresponds to the practice already implemented in equation 9 but applied to relative changes rather than absolute numbers.

<u>Result 1.</u> The left term in the last form of (11) can be presented as follows:

$$ln\left(\frac{V_1^*}{V_0}\right) = ln\left(\frac{\sum_{i=1}^{n} p_{1,i}q_{1,i}^*}{\sum_{i=1}^{n} p_{0,i}q_{0,i}}\right)$$

$$= ln\left({P_0^1}_{Paasche} \times {Q_0^1}_{Laspeyres}\right) = ln\left(\frac{\sum_{i=1}^{n} p_{1,i}q_{1,i}^*}{\sum_{i=1}^{n} p_{0,i}q_{1,i}^*} \times \frac{\sum_{i=1}^{n} p_{0,i}q_{1,i}^*}{\sum_{i=1}^{n} p_{0,i}q_{0,i}}\right) \tag{12}$$

$$= ln\left({P_0^1}_{Laspeyres} \times {Q_0^1}_{Paasche}\right) = ln\left(\frac{\sum_{i=1}^{n} p_{1,i}q_{0,i}}{\sum_{i=1}^{n} p_{0,i}q_{0,i}} \times \frac{\sum_{i=1}^{n} p_{1,i}q_{1,i}^*}{\sum_{i=1}^{n} p_{1,i}q_{0,i}}\right)$$

Hence, two alternative formulations for the Metla Index can be derived in Laspeyres-Paasche space. Although **this is not surprising in the context of the index number theory**, it should be noted that the formulations are equally correct as index theoretical formulations for the Metla Index. Whilst there is virtually no existing discussion related to the choice (nor can it be concluded whether the alternative formulations have been previously identified), the FFRI has adopted the latter decomposition.

Concerning the latter decomposition of FFRI, it can further be illustrated that the ratios are actually weighted with values shares. We elaborate further the latter decomposition in (12) adopted by FFRI flowingly:

$$
\begin{aligned}
ln\left(\frac{V_1^*}{V_0}\right) &= \ln\left(\frac{\sum_{i=1}^n p_{1,i}q_{0,i}}{\sum_{i=1}^n p_{0,i}q_{0,i}}\right) + \ln\left(\frac{\sum_{i=1}^n p_{1i}q_{1,i}^*}{\sum_{i=1}^n p_{1i}q_{0,i}}\right) \\
&= \ln\left(\frac{\sum_{i=1}^n p_{1,i}q_{0,i}}{\sum_{i=1}^n p_{0,i}q_{0,i}}\right) + \ln\left(\frac{1}{\frac{\sum_{i=1}^n p_{1i}q_{0,i}}{\sum_{i=1}^n p_{1i}q_{1,i}^*}}\right) \\
&= \ln\frac{\sum_{i=1}^n p_{0,i}q_{0,i}\left(\frac{p_{1,i}}{p_{0,i}}\right)}{\sum_{i=0}^n p_{0,i}q_{0,i}} + \ln\left(\frac{1}{\frac{\sum_{i=1}^n p_{1i}q_{1,i}^*\left(\frac{q_{0,i}}{q_{1,i}^*}\right)}{\sum_{i=1}^n p_{1i}q_{1,i}^*}}\right) \qquad (13) \\
&= \ln\sum_{i=1}^n \frac{p_{0,i}q_{0,i}}{\sum_{i=0}^n p_{0,i}q_{0,i}}\left(\frac{p_{1,i}}{p_{0,i}}\right) + \ln\left[\sum_{i=1}^n \frac{p_{1i}q_{1,i}^*}{\sum_{i=1}^n p_{1i}q_{1,i}^*}\left(\frac{q_{i,1}^*}{q_{0,i}}\right)^{-1}\right]^{-1}
\end{aligned}
$$

Thus, the Laspeyres form is the simple arithmetic weighted average ratio whereas the Paasche is the harmonic one. It turns out that this Laspeyres-Paasche pair is just one possible solution to the value ratio $\ln\left(\frac{V_1^*}{V_0}\right)$ [17].

[17]Price and quantity index pairs that are exact with the value ratio can simply be produced by following relation $\frac{V_1}{V_0} = P_0^1 Q_0^1$ between value, price and quantity index. See Pursiainen (2008b) for a presentation of additive value decompositions and quasilinear means.

From the equations we can see that Laspeyres' indices weight the ratio with base period's value shares whereas Paasche's weights them with observation period's shares. It is essential to understand that because of the opposite weighting, the Laspeyres' and Paasche's indices produce different results. The two indices form the opposite tips of the index fork, the two extremes, and the price and quantity weights have to be the opposites of each other (cross formulated) for the mean at the aggregate value level to remain intact and the decomposition to be mathematically exact (Pursiainen 2008a, b).

5. Decomposing the Metla Volume Index

The official and published Metla Index presents an *approximate* decomposition of the volume index of equation (13) into contributions of harvesting and the net biological growth ($\gamma_{1,i}$) as follows (see Penttinen and Lausti 2004):

$$\begin{aligned} \ln\left(\frac{\sum_{i=1}^{n} p_{1i} q_{1,i}^{*}}{\sum_{i=1}^{n} p_{1i} q_{0,i}}\right) &= \ln\left(1+\frac{\sum_{i=1}^{n} p_{1,i}\left(h_{1,i}+\gamma_{1,i}\right)}{\sum_{i=1}^{n} p_{1,i} q_{0,i}}\right) \\ &\cong \ln\left(1+\frac{\sum_{i=1}^{n} p_{1,i}\, h_{1,i}}{\sum_{i=1}^{n} p_{1,i} q_{0,i}}\right)+\ln\left(1+\frac{\sum_{i=1}^{n} p_{1,i}\, \gamma_{1,i}}{\sum_{i=1}^{n} p_{1,i} q_{0,i}}\right) \end{aligned} \tag{14}$$

In view of the approximation, Lausti and Penttinen (1998) acknowledge the presence of the following additive correction term to offset the bias:

$$\ln\left[1-\frac{\frac{\sum_{i=1}^{n} h_{1,i}}{\sum_{i=1}^{n} q_{0,i}} \times \frac{\sum_{i=1}^{n} \gamma_{1,i}}{\sum_{i=1}^{n} q_{0,i}}}{\left(1+\frac{\sum_{i=1}^{n} h_{1,i}}{\sum_{i=1}^{n} q_{0,i}}\right)\left(1+\frac{\sum_{i=1}^{n} \gamma_{1,i}}{\sum_{i=1}^{n} q_{0,i}}\right)}\right] \tag{15}$$

Thus, the Metla Index does not perform exact decomposition.

In the following, we will explore the exact decomposition of the Metla Index and derive solutions to establish an analytically *exact* formula for harvesting volume change and net biological growth. It appears that four possible and mutually different formulations exist for both the harvesting index as well as the net biological growth due to a cross term which cannot be eliminated.

***Result 2*. Exact decomposition of the Metla Volume Index:**

$$
\begin{aligned}
&ln\left(\frac{\sum_{i=1}^{n} p_{1i} q_{1,i}^{*}}{\sum_{i=1}^{n} p_{1i} q_{0,i}}\right) \\
&= ln\left(1+\frac{\sum_{i=1}^{n} p_{1,i}\,\gamma_{1,i}}{\sum_{i=1}^{n} p_{1,i} q_{0,i}}\right) + ln\left(1+\frac{\sum_{i=1}^{n} p_{1,i}\,h_{1,i}}{\sum_{i=1}^{n} p_{1,i} q_{0,i}+\sum_{i=1}^{n} p_{1,i}\,\gamma_{1,i}}\right) \\
&= ln\left(1+\frac{\sum_{i=1}^{n} p_{1,i}\,\gamma_{1,i}}{\sum_{i=1}^{n} p_{1,i} q_{0,i}+\sum_{i=1}^{n} p_{1,i}\,h_{1,i}}\right) + ln\left(1+\frac{\sum_{i=1}^{n} p_{1,i}\,h_{1,i}}{\sum_{i=1}^{n} p_{1,i} q_{0,i}}\right)
\end{aligned} \tag{16}
$$

i.e. there are two equally valid exact decompositions for the Metla Volume Index.

Proof: *Paasche's net growth index and modified harvest index derived from the volume index.*

$$
\begin{aligned}
ln\left(\frac{\sum_{i=1}^{n} p_{1i} q_{1,i}^{*}}{\sum_{i=1}^{n} p_{1i} q_{0,i}}\right) &= ln\left(1+\frac{\sum_{i=1}^{n} p_{1,i}\left(h_{1,i}+\gamma_{1,i}\right)}{\sum_{i=1}^{n} p_{1,i} q_{0,i}}\right) \\
&= ln\left(1+\frac{\sum_{i=1}^{n} p_{1,i}\,h_{1,i}}{\sum_{i=1}^{n} p_{1,i} q_{0,i}}+\frac{\sum_{i=1}^{n} p_{1,i}\,\gamma_{1,i}}{\sum_{i=1}^{n} p_{1,i} q_{0,i}}\right)
\end{aligned} \tag{17}
$$

To separate the two components the net growth index is taken as a common term.

$$
\begin{aligned}
&ln\left[\frac{\sum_{i=1}^{n} p_{1,i} q_{0,i}+\sum_{i=1}^{n} p_{1,i}\,\gamma_{1,i}}{\sum_{i=1}^{n} p_{1,i} q_{0,i}} \times \left(1+\frac{\sum_{i=1}^{n} p_{1,i}\,h_{1,i} \times \sum_{i=1}^{n} p_{1,i} q_{0,i}}{\sum_{i=1}^{n} p_{1,i} q_{0,i} \times \left(\sum_{i=1}^{n} p_{1,i} q_{0,i}+\sum_{i=1}^{n} p_{1,i}\,\gamma_{1,i}\right)}\right)\right] \\
&= ln\left[\left(1+\frac{\sum_{i=1}^{n} p_{1,i}\,\gamma_{1,i}}{\sum_{i=1}^{n} p_{1,i} q_{0,i}}\right) \times \left(1+\frac{\sum_{i=1}^{n} p_{1,i}\,h_{1,i}}{\sum_{a=1}^{n} p_{1,i} q_{0,i}+\sum_{a=1}^{n} p_{1,i}\,\gamma_{1,i}}\right)\right] \\
&= ln\left(1+\frac{\sum_{i=1}^{n} p_{1,i}\,\gamma_{1,i}}{\sum_{i=1}^{n} p_{1,i} q_{0,i}}\right) + ln\left(1+\frac{\sum_{i=1}^{n} p_{1,i}\,h_{1,i}}{\sum_{i=1}^{n} p_{1,i} q_{0,i}+\sum_{i=1}^{n} p_{1,i}\,\gamma_{1,i}}\right)
\end{aligned} \tag{18}
$$

The proof for the latter part of Result 2 is straightforward by applying the harvest index as a common term in (17) above.

Recall that in (12) an alternative feasible choice for volume index, from the viewpoint of index number theory, could have been as well the Laspeyres form $\frac{\sum_{i=1}^{n} p_{0,i} q_{1,i}^{*}}{\sum_{i=1}^{n} p_{0,i} q_{0,i}}$. Had we adopted this form of a volume index, we can establish again two additional, equally valid formulations for the harvesting volume index and net biological growth volume index as follows.

<u>Result 3</u>. Exact decomposition of the Laspeyres volume index in the context of the Metla Index:

$$
\begin{aligned}
& ln\left(\frac{\sum_{i=1}^{n} p_{0,i} q_{1,i}^{*}}{\sum_{i=1}^{n} p_{0,i} q_{0,i}}\right) \\
&= ln\left(1+\frac{\sum_{i=1}^{n} p_{0,i}\, \gamma_{1,i}}{\sum_{i=1}^{n} p_{0,i} q_{0,i}}\right)+ln\left(1+\frac{\sum_{i=1}^{n} p_{0,i}\, h_{1,i}}{\sum_{i=1}^{n} p_{0,i} q_{0,i}+\sum_{i=1}^{n} p_{0,i}\, \gamma_{1,i}}\right) \\
&= ln\left(1+\frac{\sum_{i=1}^{n} p_{0,i}\, \gamma_{1,i}}{\sum_{i=1}^{n} p_{0,i} q_{0,i}+\sum_{i=1}^{n} p_{0,i}\, h_{1,i}}\right)+ln\left(1+\frac{\sum_{i=1}^{n} p_{0,i}\, h_{1,i}}{\sum_{i=1}^{n} p_{0,i} q_{0,i}}\right)
\end{aligned}
\tag{19}
$$

Proof: *Follows the method applied for Result 2 above.*

Figure 4 summarizes the four possible exact volume index solutions derived above in the context of Result 1, Result 2 and Result 3.

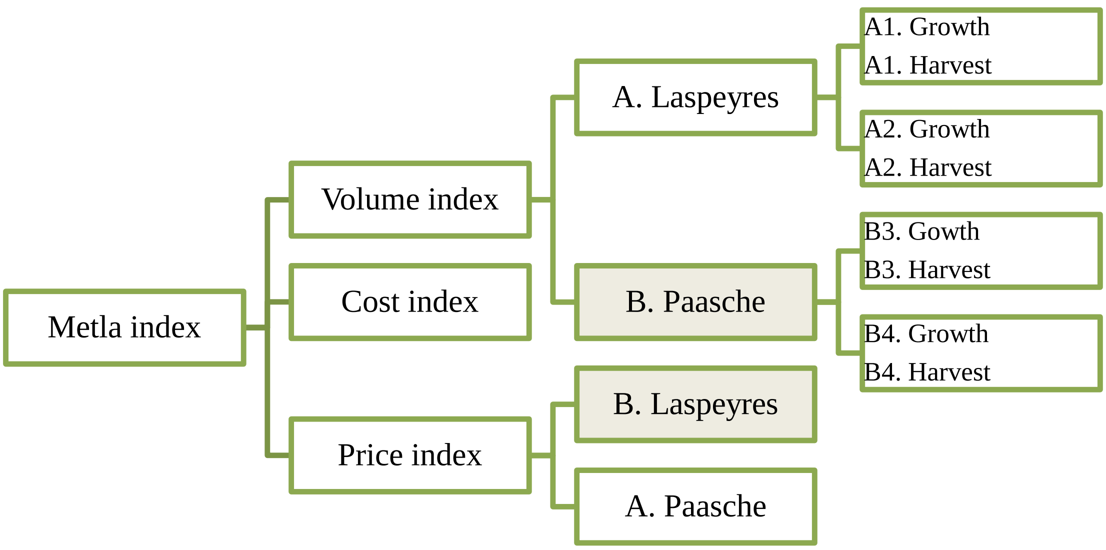

Figure 4. The four exact and equally valid decompositions for the Metla index.

Hence, the methodology employed within the Metla Index leads to four different but equally valid formulations both for the harvesting volume index and the net biological growth index. As noted above, harvesting is comparable to the *new good* and there is no unambiguous indicator for harvesting volume change. In addition, there are two valid solutions for price index based on Laspeyres or Paasche. Also, the space of "acceptable" solutions could be continued by establishing suitable approximations such as the one applied by FFRI.

Ignoring for the approximation applied, the Metla Index correctly applies different Laspeyres-Paasche index formulas with the product of the volume and price indices matching exactly the value ratio. As discussed in the context of (13), while representing different functional forms, Laspeyres and Paasche form an index pair producing quasilinear mean equaling precisely the initial value ratio, in our case TCP (Pursiainen 2008b; see also Vartia 1978, Vartia and Vartia 1984). However, it is well known that the group of set indices neutralizing each other around the gravity point is large and that by selecting a suitable pair of indices for volume and price, respectively, an exact match to value ratio can be established. Hence, Laspeyres and Paasche is not the only pair of indices which could be successfully applied for the productivity measurement of timberland capital with the exact match to the value ratio. On the other hand, each pair of suitably selected indices at the opposite tips of the index fork would yield again a different outcome for the volume and price indices of interest.

If the only guiding criteria for the index formulas to be applied is to match the value ratio indicated by TCP measure, it is difficult exhaustively to agree about the "correct" index formulas to be applied. Arbitrariness in the choice of the index formula tend to affect the mutual relationships of price and volume indices as well as the sub-volume indices. Hence, one cannot escape from the fact that *consistency in aggregation* (Definition 1) is a key criteria for the measurement of productivity of timberland capital.

6. Timberland productivity with consistent aggregation

As shown in the previous sections, the index number formula for timberland capital productivity has to aggregate over a set of timber grades in several dimensions including harvesting, net biological growth, total biological growth and price change. In order to be consistent in the context of Definition 1 above, the aggregate based on sub-sets has to be established based on one and the same index number formula and it has be equal to TCP (Definition 2 above), i.e. the value ratio defined as a measure for timberland capital productivity. In harmony with Definition 1, we introduce in the following a Montgomery-Vartia type of TCP index allowing exact mathematical decomposition of the aggregate index into sub-indices that all follow the same form.

Result 4. ***Montgomery-Vartia TCP Index***

$$TCP_{MV} = ln\left(\frac{V_1^* - C_1}{V_0}\right)$$
$$= \sum_{i=1}^{n} \frac{L\left(p_{1,i}q_{1,i}^* \text{ , } p_{0,i}q_{0,i}\right)}{L(P_1Q_1^*, P_0Q_0)} ln\left(\frac{p_{1,i}q_{1,i}^*}{p_{0,i}q_{0,i}}\right) + ln\left(1 - \frac{C_1}{\sum_{i=1}^{n} p_{1,i}q_{1,i}^*}\right) \tag{20}$$

where the function L is defined as $L(a, b) = \frac{a-b}{ln\left(\frac{a}{b}\right)}$ *(Logarithmic mean or Vartia mean)*

The subsequent price and volume indices are (the first level decomposition):

$$P_0^1{}_{MV} = \sum_{i=1}^{n} \frac{L\left(p_{1,i}q_{1,i}^* \text{ , } p_{0,i}q_{0,i}\right)}{L(P_1Q_1^*, P_0Q_0)} ln\left(\frac{p_{1,i}}{p_{0,i}}\right) \tag{21}$$

$$Q_0^1{}_{MV} = \sum_{i=1}^{n} \frac{L\left(p_{1,i}q_{1,i}^* \text{ , } p_{0,i}q_{0,i}\right)}{L(P_1Q_1^*, P_0Q_0)} ln\left(\frac{q_{1,i}^*}{q_{0,i}}\right) \tag{22}$$

$$C = ln\left(1 - \frac{C_1}{\sum_{i=1}^{n} p_{1,i}q_{1,i}^*}\right) \tag{23}$$

and the volume index decomposition (the second level decomposition):

$$Net\ Growth\ index_{MV} = \sum_{i=1}^{n} \frac{L\left(p_{1,i}q_{1,i} \text{ , } p_{0,i}q_{0,i}\right)}{L(P_1Q_1, P_0Q_0)} ln\left(\frac{q_{1,i}}{q_{0,i}}\right) \tag{24}$$

$$Harvest\ index_{MV} = \sum_{i=1}^{n} \frac{L\left(p_{1,i}q_{1,i}^* \text{ , } p_{0,i}q_{0,i}\right)}{L(P_1Q_1^*, P_0Q_0)} ln\left(\frac{q_{1,i}^*}{q_{0,i}}\right) - \sum_{i=1}^{n} \frac{L\left(p_{1,i}q_{1,i} \text{ , } p_{0,i}q_{0,i}\right)}{L(P_1Q_1, P_0Q_0)} ln\left(\frac{q_{1,i}}{q_{0,i}}\right) \tag{25}$$

Proof:

Denoting $v_{t,i} = p_{t,i} \times q_{t,i}$, we get

$$ln\left(\frac{V_1^* - C_1}{V_0}\right) = ln\left[\frac{V_1^*}{V_0} \times \left(1 - \frac{C_1}{V_1^*}\right)\right] = \frac{\sum_{i=1}^{n}(v_{1,i}^* - v_{0,i})}{L(V_1^*, V_0)} + ln\left(1 - \frac{C_1}{V_1^*}\right) \qquad (26)$$

since the logarithmic mean is a relative change with log-change as a base $L\left(v_{1,i}^*, v_{0,i}\right) = \frac{v_{1,i}^* - v_{0,i}}{ln\left(\frac{v_{1,i}^*}{v_{0,i}}\right)}$. By rearranging we get $(v_{1,i}^* - v_{0,i}) = L\left(v_{1,i}^*, v_{0,i}\right) ln\left(\frac{v_{1,i}^*}{v_{0,i}}\right)$. Implementing this and by further splitting the productivity index into product of price and quantity, the original equation can be written as follows:

$$\begin{aligned} TCP_{MV} &= \frac{\sum_{i=1}^{n} L\left(v_{1,i}^*, v_{0,i}\right) ln\left(\frac{v_{1,i}^*}{v_{0,i}}\right)}{L(V_1^*, V_0)} - ln\left(1 - \frac{C_1}{\sum_{i=1}^{n} p_{1,i} q_{1,i}^*}\right) \\ &= \frac{\sum_{i=1}^{n} L\left(p_{1,i} q_{1,i}^*, \, p_{0,i} q_{0,i}\right) ln\left(\frac{p_{1,i} q_{1,i}^*}{p_{0,i} q_{0,i}}\right)}{L(P_1 Q_1^*, P_0 Q_0)} - ln\left(1 - \frac{C_1}{\sum_{i=1}^{n} p_{1,i} q_{1,i}^*}\right) \end{aligned} \qquad (27)$$

$$= \sum_{i=1}^{n} \frac{L\left(p_{1,i} q_{1,i}^*, \, p_{0,i} q_{0,i}\right)}{L(P_1 Q_1^*, P_0 Q_0)} ln\left(\frac{p_{1,i}}{p_{0,i}}\right) + \sum_{i=1}^{n} \frac{L\left(p_{1,i} q_{1,i}^*, \, p_{0,i} q_{0,i}\right)}{L(P_1 Q_1^*, P_0 Q_0)} ln\left(\frac{q_{1,i}^*}{q_{0,i}}\right) - ln\left(1 - \frac{C_1}{\sum_{i=1}^{n} p_{1,i} q_{1,i}^*}\right) \qquad (28)$$

In view of the fact that the standing stock volume $q_{1,i}$ is an observable variable we can explicitly decompose the index for the volume indicating the net biological growth:

$$Net\ Growth\ index_{MV} = \sum_{i=1}^{n} \frac{L\left(p_{1,i} q_{1,i}, \, p_{0,i} q_{0,i}\right)}{L(P_1 Q_1, P_0 Q_0)} ln\left(\frac{q_{1,i}}{q_{0,i}}\right) \qquad (29)$$

The second level decomposition faces the new good problem related harvest quantity (h). We utilize here the index's proven property of being consistency in aggregation and construct the harvesting index by following the same Montgomery-Vartia principle as follows by subtracting the net biological growth index from the aggregate volume index:

$$Harvest\ index_{MV} = \sum_{i=1}^{n} \frac{L\left(p_{1,i} q_{1,i}^*, \, p_{0,i} q_{0,i}\right)}{L(P_1 Q_1^*, P_0 Q_0)} ln\left(\frac{q_{1,i}^*}{q_{0,i}}\right) - \sum_{i=1}^{n} \frac{L\left(p_{1,i} q_{1,i}, \, p_{0,i} q_{0,i}\right)}{L(P_1 Q_1, P_0 Q_0)} ln\left(\frac{q_{1,i}}{q_{0,i}}\right) \qquad (30)$$

To note for the exact additivity we can write:

$$
\begin{aligned}
&Timberland\ capital\ productivity\ index_{MV} \\
&\qquad = Price\ index_{MV} + Volume\ index_{MV} + Cost\ index \\
&\qquad = Price\ index_{MV} + \ Net\ Growth\ index_{MV} + Harvest\ index_{MV} + Cost\ index
\end{aligned}
$$

Result 4 thus indicates that timberland capital productivity can be broken into its components with mathematically exact match by applying systematically the Montgomery-Vartia formula for all sub-indices at all stages of the decompositions[18]. Hence, there is no scope for arbitrary choice of index numbers in the sense discussed in the previous sections. Nor there is scope for speculation regarding the mutual relationships between the price and volume indices or between the volume sub-indices on the other hand. Furthermore, no approximations are required.

It should be noted that the criteria of consistency in aggregation and the Montgomery-Vartia formula generate exactly one solution for each index at each level of decomposition. This feature greatly increases the usefulness of the results in economic analysis and for forest policy purposes.

[18] Note that Stuvel index is also consistent in aggregation, while being based on Laspeyres-Paasche (Stuvel 1957):

$$\mathrm{P}^1_{0\,\mathrm{Stuvel}} = \frac{\mathrm{P}^1_{0\mathrm{L}} - \mathrm{Q}^1_{0\mathrm{L}}}{2} + \sqrt{\left(\frac{\mathrm{P}^1_{0\mathrm{L}} - \mathrm{Q}^1_{0\mathrm{L}}}{2}\right)^2 + \frac{\mathrm{V}_1}{\mathrm{V}_0}}, \quad \mathrm{Q}^1_{0\,\mathrm{Stuvel}} = \frac{\mathrm{Q}^1_{0\mathrm{L}} - \mathrm{P}^1_{0\mathrm{L}}}{2} + \sqrt{\left(\frac{\mathrm{Q}^1_{0\mathrm{L}} - \mathrm{P}^1_{0\mathrm{L}}}{2}\right)^2 + \frac{\mathrm{V}_1}{\mathrm{V}_0}}$$

7. Empirical analysis

The same TCP measure will be in the following applied in the context of different index number formulas as discussed above. The different decomposition practices will yield different sub-indices and therefore different index numbers for the same phenomena will result. To illustrate the how the different index formulas behave in practice we obtained timber stock and trade statistics for the Finnish private forests' and calculated indices for 2007–2011 with the different formulas considered here. The data series should be considered illustrative due to simplifications and interpolation. The data is listed in appendix I and is based on statistics obtained from the Finnish Forest Research Institute's statistical service MetINFO (Finnish Forest Research Institute. 2013). The same data is inserted into all the formulas.

For sole economic price and quantity indices the superlative indices are often regarded as ideal[19]. Here the Fisher's Ideal index is chosen to represent the superlative index class. Fisher's Ideal is calculated as the geometric mean of the Laspeyres' and Paasche's indices[20]. Our desideratum, consistency in aggregation, is selected to be the benchmark and thus the different indices are compared to the Montgomery-Vartia. Also the often ignored but consistent Stuvel's indices, for which we do not find many reflections in the literature, are plotted. The differences are reported in basis points transformed from log-percentage scale through Euler's number[21].

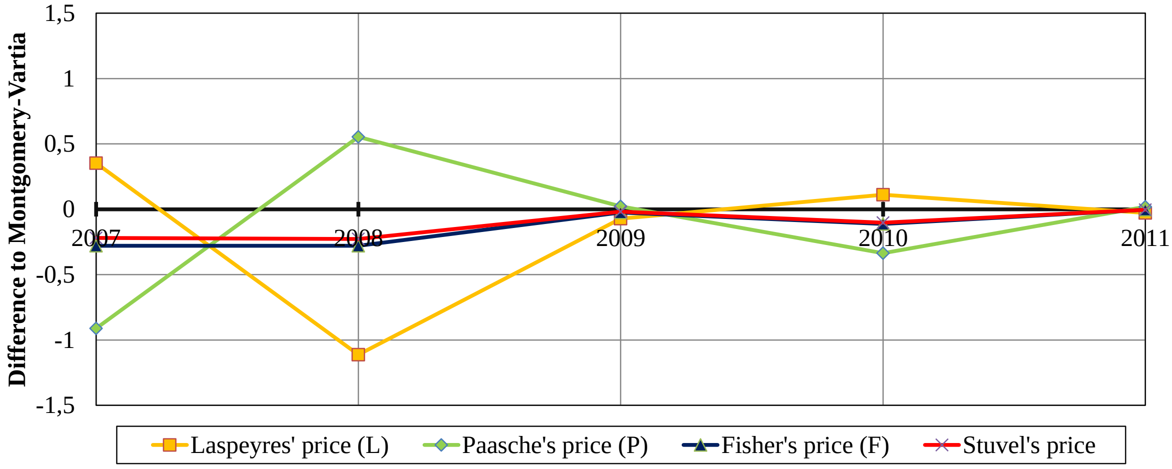

Figure 5. Price index comparison with Finnish private forests' return series for 2007-2011, (bp).

[19] See IMF 2004, p. 5, OECD 2001, p. 83, ILO 2004, ch. 9 p. 20.

[20] $P^1_{0\ \text{Fisher}} = \sqrt{\frac{\Sigma(p_{1,i}q_{0,i})}{\Sigma(p_{0,i}q_{0,i})} \times \frac{\Sigma(p_{1,i}q_{1,i})}{\Sigma(p_{0,i}q_{1,i})}}, \quad Q^1_{0\ \text{Fisher}} = \sqrt{\frac{\Sigma(p_{0,i}q_{1,i})}{\Sigma(p_{0,i}q_{0,i})} \times \frac{\Sigma(p_{1,i}q_{1,i})}{\Sigma(p_{1,i}q_{0,i})}}$

[21] See appendix II for a full list of the calculated index numbers for the return series.

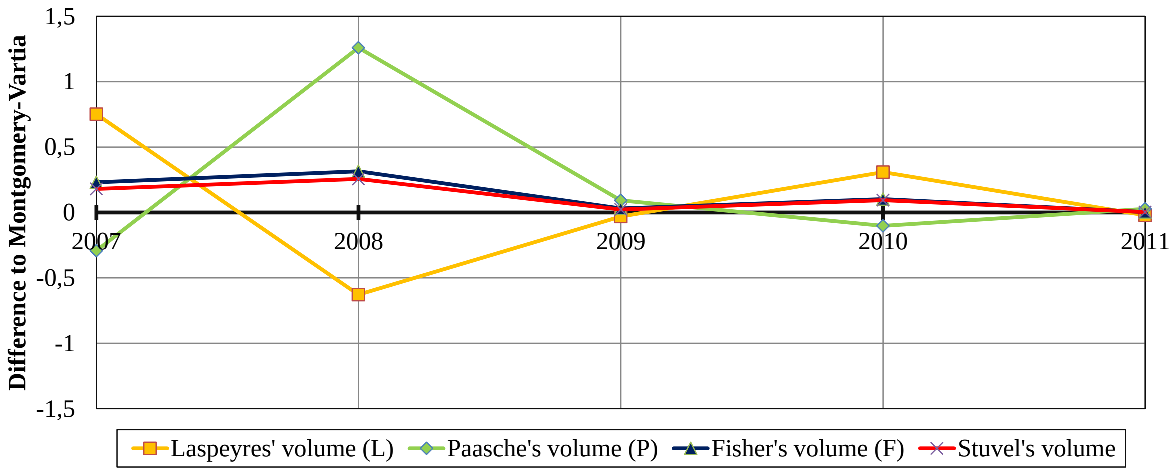

Figure 6. Quantity index comparison with Finnish private forests' return series for 2007-2011, (bp).

The yellow (squares) and green (diamonds) marks are the alternative index numbers the decomposition practice selected by the Finnish Forests Research Institute yields. We can see the differences of the Laspayres' and Paasche's indices and that they really are the outer edges of the index fork while the superlative Fisher gather somewhere around the imagined center of gravity. The order of the base and observation period weighted indices in a given period is dependent of the correlation between price and quantity which translates into elasticity in economics. While the Paasche's and Laspeyres' indices bounce up and down it can be perceived that the difference between the Fisher's Ideal and the Montgomery-Vartia is more stable. Being based on Laspeyres-Paasche to calculate an average index, Stuvel's indices follow closely the geometric mean of the two indices – the Fisher's Ideal. Balk (2008) notes that "though the Stuvel indices are consistent-in-aggregation, they are decomposition-resistant..."

When the harvest indices[22] derived by Results 2 and 3, are plotted we can see in figure 7 that even on further decompositions the Montgomery-Vartia is still in the middle of the other two possible subindices. A genuine Fisher's Ideal index cannot be calculated for the derived exact harvest indices because they do not follow the Paasche and Laspeyres forms anymore. For the same reason one faces the same trade-off between inexact but consistent and exact but inconsistent index formulas than the Metla index as presented in Result 3 when formulating the harvest and net growth indices according to the Stuvel's formula. The index is not calculated for harvests.

[22] Paasche derived: $\ln\left(1+\frac{\sum_{i=1}^{n} p_{1,i}h_{1,i}}{\sum_{i=1}^{n} p_{1,i}q_{0,i}+\sum_{i=1}^{n} p_{1,i}\gamma_{1,i}}\right)$ and Laspeyres derived: $\ln\left(1+\frac{\sum_{i=1}^{n} p_{0,i}h_{1,i}}{\sum_{i=1}^{n} p_{0,i}q_{0,i}+\sum_{i=1}^{n} p_{0,i}\gamma_{1,i}}\right)$

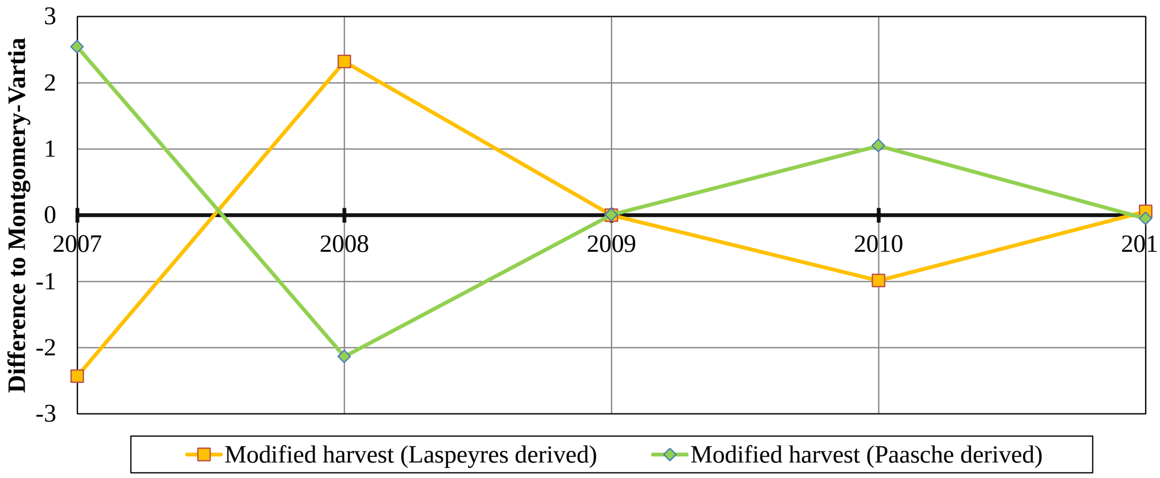

Figure 7. Harvest index comparison with Finnish private forests' return series for 2007-2011, (bp).

The differences and the advantages of the Montgomery-Vartia index formula become clear at the decomposition level. When the Laspeyres and Paasche indices always forms the outer edges of the index fork, the Montgomery-Vartia yields index numbers that do not favor any period over another. The figure 8 also shows that even if the spread between Paasche and Laspeyres widens the Montgomery-Vartia will remain in the middle and close to Fisher. The pseudo-superlative Montgomery-Vartia index approximates closely the ideal superlative index.

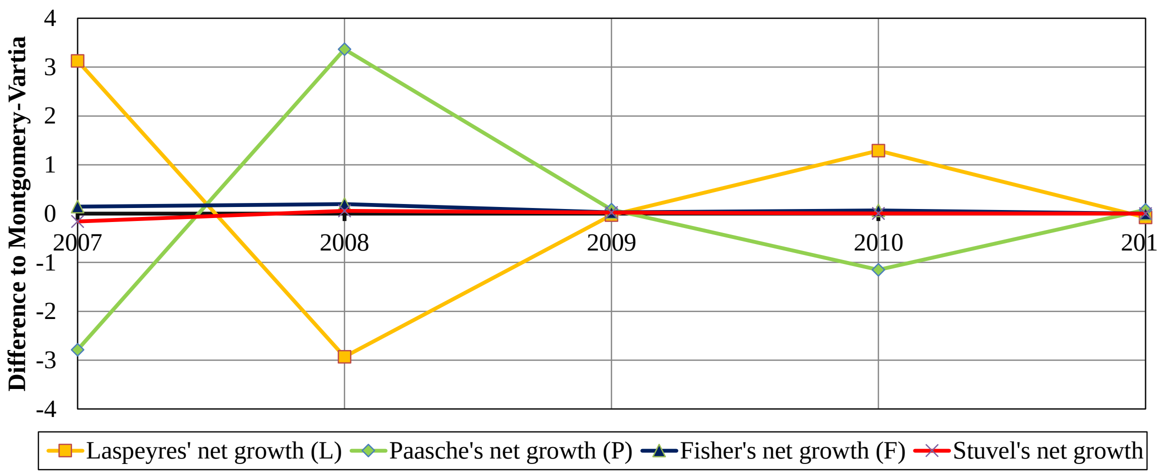

Figure 8. Net growth index comparison with Finnish private forests' return series for 2007-2011, (bp).

8. Discussion and recommendations

The measurement of productivity for timberland capital has been analyzed above based on index number theory and predominantly from the perspective of consistency in aggregation (Definition 1). Whilst there are a wide range of desideratums to be fulfilled by an index number formula, we note that neither the desideratum of consistency in aggregation nor further index theory implications have been discussed in the context of TCP measurement. Due to the theoretical attractiveness as well as the importance of the concept in pragmatic measurement problems, the consistency in aggregation which was initially introduced by Vartia (1976) has been subject to extensive studies since the 1970s (e.g. Diewert (1978), Balk (1996), Blackorby and Primont (1980), Stuvel (1989), Pursiainen (2005) and Pursiainen (2008a, b)).

TCP measurement appears to form an advanced problem in the light of index number theory. In above, we first establish a conceptual framework for TCP (Definition 2). Whilst the previous studies on TCP do not make reference to index number theory, we show that the established TCP concept lends itself strongly to classical index number theory issues which need to be addressed in order to be able to establish an appropriate measurement model for TCP and its components.

Second, we examine the Metla Index in the established framework. Although regularly published by FFRI, it seems that the theoretical foundations of the Metla Index have not been exhaustively discussed. In the Laspeyres-Paasche space applied by the Metla Index we identify several parallel and equally valid indices which all have the same theoretical justification. Yet only one index out of the several possible indices is being published by FFRI. Thus, while the timberland profitability is frequently reported by FFRI by splitting the profitability into components of volume factors and price change there would be a call for a careful analysis of the measurement consistency. Our argument is not related to the concept of TCP applied but only to the decomposition to price and volume indices, the magnitude of which could be affected by applying arbitrarily alternative but equally valid index formulas. On the other hand, the volume and price indices are gradually becoming increasingly important benchmarks in forest policies, forest entrepreneurs and timberland investors.

From the viewpoint of the economic theory, it might also be worthwhile to note that the Metla Index imposes implicitly a rather strong constraint on the technology. Being based on Laspeyres or Paasche indices, the Metla Index imply either a Leontief production function for which there is no substitution or a linear production function in which all inputs are perfect substitutes. The least restricting technology would be implied by the so called superlative indices which are exact with flexible functional forms approximating arbitrary twice differentiable linearly homogenous functions to the second order. Whilst we advocate here the descriptive and the mathematically attractive property of the Montgomery-Vartia index of being consistent in aggregation, it is noted that the Montgomery Vartia index is also classified as a pseudo-superlative index and that it approximates any superlative index to the second order around an equal price and quantity point.

We introduce a measure for the timberland capital productivity based on Montgomery-Vartia index. We show that our measure for TCP and its volume and price components can be derived as an exact mathematical solution to the logarithm of the value ratio indicated by the defined TCP. Due to the well-established consistency in aggregation -nature of the Montgomery-Vartia index, our TCP measure can be derived by an additive calculus of underlying volume and price changes by applying only one and the same functional form for the aggregator. Hence, the derived volume and price indices are not exposed to arbitrary choice of alternative index number formulas.

Our empirical analysis confirms the differences between the examined index number formulas in TCP measurement. As can be expected, the observed differences match the predictions of the index number theory. On the other hand, it is normal for empirical analysis regarding index numbers that the nature of data affects the magnitude of the differences observed. Empirical price and quantity data relatively rarely exhibits dramatic changes in the analysis of sequential periods. However, index number calculations such as TCP measurement should be able to retain consistency also during the peak periods of economic high and low cycles when the demand for the calculated results tends to be at highest.

At least the following avenues for future research can be identified. The TCP definition in the current study is based on the liquidation value of timberland. It could be useful to consider alternative definitions for timberland value incorporating also some ideas discussed in section 2. The value of the land is clearly an issue which would need to be addressed more thoroughly in TCP analysis. It might also be worthwhile to elaborate a net present value-based version of timberland value for TCP measurement which would imply an analysis of the role of a discount rate in defining TCP and its components.

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Appendices

Appendix I. Input parameters of the empirical analysis.

m^3		q		γ		h		q*	
		Logs	**Pulpwood**	**Logs**	**Pulpwood**	**Logs**	**Pulpwood**	**Logs**	**Pulpwood**
2006	Pine	296545000	639062000	6965573	21951427	7092000	8343000		
2006	Spruce	269861000	313972000	1961775	8027225	11029000	7087000		
2006	Birch	35739000	281614000	1559747	9814797	866000	4898000		
2007	Pine	301488645	660059355	4943645	20997355	9248000	9586000	310736645	669645355
2007	Spruce	269988389	321658611	127389	7686611	13070000	7668000	283058389	329326611
2007	Birch	37028751	290640300	1289751	9026300	1128000	5638000	38156751	296278300
2008	Pine	308153973	680537027	6665328	20477672	7384000	10281000	315537973	690818027
2008	Spruce	273872720	330639280	3884331	8980669	9285000	6709000	283157720	337348280
2008	Birch	38413725	303362327	1384974	12722026	1038000	6296000	39451725	309658327
2009	Pine	316975674	703635326	8821701	23098299	5144000	7744000	322119674	711379326
2009	Spruce	279737221	340830779	5864501	10191499	7210000	5593000	286947221	346423779
2009	Birch	40138574	317018478	1724849	13656151	685000	5375000	40823574	322393478
2010	Pine	323950748	725136252	6975074	21500926	7025000	9577000	330975748	734713252
2010	Spruce	284141994	351355006	4404773	10524227	9810000	6795000	293951994	358150006
2010	Birch	41931613	331300439	1793039	14281961	811000	6285000	42742613	337585439
2011	Pine	330641520	746804480	6690772	21668228	7229000	9490000	337870520	756294480
2011	Spruce	288223406	362137594	4081411	10782589	10018000	6652000	298241406	368789594
2011	Birch	43633812	345803240	1702199	14502801	901000	6065000	44534812	351868240

		p (EUR/m^3)		C_1 (EUR)
		Logs	**Pulpwood**	**Costs p.a.**
2006	Pine	48,84	13,26	314019000
	Spruce	50,4	21,84	
	Birch	43,36	12,75	
2007	Pine	65,64	16,32	330388000
	Spruce	67,25	24,32	
	Birch	47,68	15,41	
2008	Pine	57,71	17,11	355072000
	Spruce	57,58	23,01	
	Birch	49,43	16,04	
2009	Pine	46,03	13,93	363400000
	Spruce	46,59	18,34	
	Birch	36,32	13,47	
2010	Pine	54,03	15,52	333405000
	Spruce	55,17	18,61	
	Birch	39,43	15,45	
2011	Pine	55,42	16,12	339010000
	Spruce	56,78	18,91	
	Birch	42,49	15,8	

Appendix II. Nominal returns for the Finnish private forests' 2007–2011.

	2007	2008	2009	2010	2011
Metla aggregate	32,68266 %	-3,38850 %	-16,19772 %	18,88840 %	7,26144 %
Metla price (L)	27,11692 %	-7,41191 %	-19,47383 %	13,96506 %	2,93107 %
Metla volume (P)	4,91372 %	4,95629 %	4,82595 %	4,91221 %	4,77743 %
Metla net growth (P)	1,72267 %	2,47343 %	2,99057 %	2,55897 %	2,45170 %
Metla harvest (P)	3,19105 %	2,48286 %	1,83539 %	2,35324 %	2,32573 %
Modified harvest (Paasche derived)	3,13701 %	2,42293 %	1,78209 %	2,29452 %	2,27007 %
Cost index	-0,51021 %	-0,58196 %	-0,72269 %	-0,56443 %	-0,54437 %
Inverse Metla					
Metla price (P)	27,10428 %	-7,39525 %	-19,47289 %	13,96059 %	2,93154 %
Metla volume (L)	4,92415 %	4,93741 %	4,82472 %	4,91632 %	4,77695 %
Metla net growth (L)	1,78183 %	2,41046 %	2,98943 %	2,58341 %	2,45015 %
Metla harvest (L)	3,14233 %	2,52695 %	1,83529 %	2,33292 %	2,32680 %
Modified harvest (Laspeyres derived)	3,08731 %	2,46748 %	1,78202 %	2,27416 %	2,27115 %
MV aggregate	32,68266 %	-3,38850 %	-16,19772 %	18,88840 %	7,26144 %
MV price	27,11339 %	-7,40079 %	-19,47312 %	13,96394 %	2,93135 %
MV volume	4,91663 %	4,94369 %	4,82502 %	4,91324 %	4,77715 %
MV net growth	1,75056 %	2,43977 %	2,98972 %	2,57049 %	2,45092 %
MV harvest	3,11160 %	2,44428 %	1,78202 %	2,28403 %	2,27058 %
Fisher's price	27,11060 %	-7,40358 %	-19,47336 %	13,96282 %	2,93131 %
Fisher's volume	4,91893 %	4,94684 %	4,82534 %	4,91427 %	4,77719 %
Fisher's net growth	1,75200 %	2,44174 %	2,99000 %	2,57116 %	2,45093 %
Stuvel's price	27,11121 %	-7,40306 %	-19,47330 %	13,96292 %	2,93131 %
Stuvel's volume	4,91844 %	4,94626 %	4,82526 %	4,91418 %	4,77719 %
Stuvel's net growth	1,74897 %	2,44035 %	2,98993 %	2,57055 %	2,45093 %